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SIMULATION OF UNDERWATER DIVERSITY ARRAY

In earlier reports [1,2] a three-dimensional underwater array which simultaneously forms and combines multiple beams in elevation, was described and analyzed.—The objective of the system is to approach a condition in which the multiple ray arrivals from a distant source are separately received and coherently combined. Because the analytical forms giving the final output signal-te-moise ratio (SNR) are too involved for direct computation, simulation experiments were carried out. Results of this work are presented below for the system described in [1, section 3, pp. 26-35] which is based on maximal ratio combining (MRC) of the multiple rays formed. At the same time certain other cases were simulated.

[3] Fred Paber, "Research in Distributed Underwater Acoustic Arrays," WTRC QPB No. 31, Povember 1979, pp. 25-35.

[2] Fred Paber and Paul Veh, "Research in Distributed Underwater Acoustic Arrays," MFRC QPR No. 32, February 1930, pp. 16-20.

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in order to see what magnitude of improvement is obtained using this method over simpler ones. Given below are results for (1) a three-dimensional array with multiple beams (branches) using only coherent phasing of branches (known as equal gain combining (EGC), (2) a three-dimensional array with multiple beams using selection of the maximum amplitude branch (known as selection combining (SC)), (3) a three-dimensional array with a single fixed focus beam, and (4) a two-dimensional array. The last one represents the original concept explored earlier and reported in [3,4,5].

The simulation results can be briefly summarized as follows. For the conditions chosen, an improvement of approximately 4 to 1 is obtained using the maximal ratio combining technique with the three-dimensional array over anything else that was done. An interesting, though expected result, was that the varlance relative to the mean of the output SNR using this best technique was much less than for the two-dimensional array. The computational model assumed ten independent ray arrivals randomly spread over \$10° in the vertical. In the twodimensional array these combine noncoherently resulting in a nearly Rayleigh fluctuation of amplitude. In the three-dimensional array the ten rays are to a large extent resolved in the separate branches. The diversity selection or coherent combining then results in a substantially smaller fluctuation.

We now describe the physical and statistical arrangements assumed. Figure 2.1 suggests the deployment of elements in a three-dimensional space. Element positions (X_n, Y_n, Z_n) , $n = 1, 2 \dots N$ were assumed independent random vectors, the number of elements N being 31 for this computation. The horizontal coordinates (X_n,Y_n) were assumed independent normally distributed random variables with zero mean and standard deviation of 50 wavelengths. This value of standard deviation implies that at 100 Hz where the wavelength is about 15 meters, about 68% of the array elements will be concentrated in a range of ± 750 meters around the center of the array. Because the array main beam was focused to look in the Y-Z plane only, the random variables X_n were not involved in the computation. The vertical

^[3] Fred Haber and William J. Graham, "Research in Distributed Underwater Acoustic Arrays," VERC QUR Mo. 24, February 1978, pp. 17-39.

^[4] Fred Haber and William J. Graham, "Research in Distributed Underwater Acoustic Arrays," VFRC QPB No. 25, May 1978, pp. 1-11.
[5] Fred Haber and William J. Graham, "Research in Distributed Underwater Acoustic

Arrays," VFRC QFR NO. 26, August 1978, pp. 29-44. .

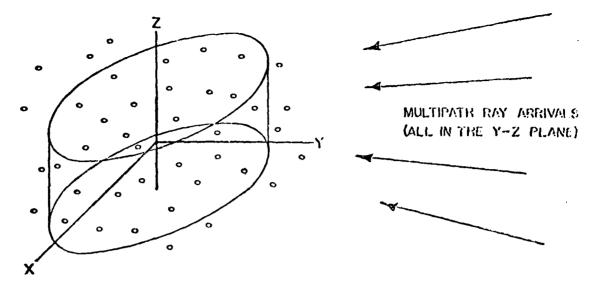


FIGURE 2.1 PHYSICAL ARRANGEMENT ASSUMED IN SIMULATION.

coordinate $Z_{\rm n}$ was assumed uniformly distributed with mean zero and range of 50 wavelengths, also. This implies a depth range of \pm 375 meters around the array center.

Ray arrivals were assumed to be in the Y-Z plane corresponding to the azimuthal angle of fecus of the array, but the rays were assumed dispersed in vertical angle. Ten rays were assumed arriving, all of equal magnitude and independent random electrical phases $\alpha_{\rm m}$, m = 1,2 ... 10, each uniformly distributed in (0, 2m). The arrival angles $\theta_{\rm m}$, measured from the vertical were assumed independent and uniformly distributed over (80°, 100°), or $\frac{1}{n}$ 10° relative to the horizontal.

Ten sets of random pairs of numbers (y_n, z_n) n = 1,2 ... 31 to represent ten possible random element positions were chosen. For each of these positions, five sets of angle pairs (θ_m, α_m) , m = 1,2 ... 10 were randomly selected.

Assuming the combining scheme of [1, section 3, Figure 3.2], the output of each branch prior to weighting and combining is given by

$$V_{i} = \sum_{m=1}^{M} \sum_{n=1}^{N} B_{m} e^{j[k(x_{m} \sin \theta_{m} \cos \phi_{m} + y_{n} \sin \theta_{m} + y_{n} \cos \theta_{m}) + \phi_{n} i} + \alpha_{m}]$$

$$= A_{j} e^{j\psi} i, \quad i = 1, 2 \dots I$$

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where (ϕ_m, θ_m) are the azimuth and elevation angles of the mth arriving ray, and (B_m, α_m) are the magnitude and electrical phase at the center of coordinates of that ray. ϕ_{ni} is the injected electrical angle to focus the array to azimuth and elevation angles (ϕ_s, θ_{si}) as shown in [1] Figure 3.2, page 31, and is given by

$$\phi_{ni} = -k[x_n \sin\theta_{si} \cos\phi_s + y_n \sin\theta_{si} \sin\phi_s + z_n \cos\theta_{si}]$$
 (2)

The subscript i identifies the beam or branch number and I is the number of branches used; here we will use I = 17 branches for reasons to be stated below. (1) and (2) are written to include rays arriving from any azimuth and elevation (ϕ_m, θ_m) and focused at any azimuth and elevation (ϕ_s, θ_{si}) . We now specialize these expressions to the case of the array focused in the Y-Z plane and ray arrivals in the Y-Z plane. Thus $\phi_m = \phi_s = 90^\circ$ and (1) and (2) reduce to

$$\mathbf{V}_{\mathbf{i}} = \sum_{m=1}^{M} \sum_{n=1}^{N} \mathbf{B}_{m} e^{\mathbf{j} \left[k y_{n} \sin \theta_{m} + z_{n} \cos \theta_{m} \right] + \phi_{n} \mathbf{i}^{+\alpha} \mathbf{n} \mathbf{i}^{-\alpha}}$$
(3)

and

$$\phi_{ni} = -k[y_n \sin\theta_{si} + y_n \cos\theta_{si}]$$
 (4)

The final signal output using MRC is given by

$$S = \sum_{i=1}^{I} |v_i|^2 = \sum_{i=1}^{I} A_i^2$$
 (5)

The mean square value of the noise at the output of the system is given by

$$N^{2} = \sum_{n=1}^{N} < n_{n}^{2} > \sum_{i=1}^{I} \sum_{j=1}^{I} A_{i} A_{j} \cos(\phi_{n,i} - \phi_{n,j} + \psi_{i} - \psi_{j})$$
 (6)

where the (A_i, ψ_i) are the measured signal amplitude and phase on the ith branch given in (1), the ϕ_{ni} are given by (4), and $\langle n_p^2 \rangle$ is the equivalent mean square value of the noise input generated by the nth sensor. In the computation $\langle n_p^2 \rangle$ was set equal to 1. Finally the computed output SNR is given by

$$SNR = \frac{S^2}{N^2}$$
 (7)

As discussed in [2] the angles θ_{si} , if set at values separated by $\frac{n\pi\lambda}{h}$, n an integer, will result in uncorrelated noise voltages at the branch output provided the elements are uniformly distributed in depth over the range (-h, h). For the geometric conditions used here it was determined that 1.15° spacing between the θ_{si} would accomplish this when h = 25 λ . We have accordingly assumed 17 beams symmetrically placed around θ = 90° at 1.15° intervals. Separation is about a beamwidth in this case and the total coverage in vertical angle is close to ±10° with respect to the horizontal.

Results of the simulation using the maximal ratio combining technique are shown in Table 1. Here we show the average and standard deviation of the SNR given by (7) over the five sets of paired values (θ_m, α_m) m = 1, 2, ... 10, for each of ten sets of position samples (y_n, z_n) n = 1, 2, ... 31. These statistics are denoted $\langle \text{SNR} \rangle_{\theta \varphi}$ and $\sigma (\text{SNR})_{\theta \varphi}$. Then the 50 results of SNR (five sets over (θ_m, α_m) times the ten sets over (y_n, z_n)) were treated as a sample of size 50 and the overall mean and standard deviation denoted $\langle \text{SNR} \rangle$ and σ respectively were determined. These were found to be $\langle \text{SNR} \rangle \stackrel{\circ}{=} 480$ and $\sigma \stackrel{\circ}{=} 182$. We point out that if a single ray were assumed to impinge on a single element the output SNR would be unity and the variance would be zero.

To determine the effect of a frequency change on the output we have assumed two different situations. In the first we assumed the angular separation between beams held at 1.15°: that is, $\theta_{\rm si}$ was held fixed at 90° ± n(1.15°), n = 0, 1, 2, ...8. The frequency was then changed by factors 1/2 and 2. These latter changes were accomplished by simply changing the values of $y_{\rm n}$ and $z_{\rm n}$ used in the previous calculation by the reciprocal of these same factors. The vertical beamwidths become narrower at the higher frequency and wider at the lower frequency but the angular spacing between beams remains unchanged. Thus the beams are not optimally spaced resulting in either uncorrelated noise in the several branches, or in nontotal coverage of the vertical range within which incoming rays are expected. Resulting these calculations are also shown in Table 1 revealing a decrease in the overall average SNR as one might expect. These results essentially show the sensitivity of the scheme to incorrect placement of the vertical beams. As we see, the effects are not overly serious, the mean output at the 1/2 and 2 times frequency points being within about 80% of the mean at the design frequency.

Position	$\sigma_{\mathbf{y}} = 50\lambda$ $h = 25\lambda$		$\sigma_y = 25\lambda$ $h = 12.5\lambda$		σ _y = 75λ h = 37.5λ	
Sample	<snr>_{θ¢}</snr>	σ(SNR) _{θφ}	<snr>_{θφ} σ(SNR)_{θφ}</snr>		<snr>_{θφ} σ(SNR)_{θφ}</snr>	
1	490	200	396	93	327	135
2	455	205	327	90	662	392
3	411	60	384	141	293	125
4	483	134	355	173	417	185
5	587	214	501	98	432	178
6	552	283	459	154	451	125
7	492	120	368	163	270	48
8	421	40	406	90	343	132
9	519	144	602	156	275	50
10	378	101	373	273	298	139
<snr></snr>	480		417		377	
σ	182		172		210	

TABLE 1 SIMULATION OF MRC ARRAY

In the second simulation of frequency effect the array focusing was held fixed for a 100 Hz sinusoid by fixing the angles ϕ_{ni} . The applied frequency was then altered to 99, 99.5 and 100.5 Hz. As a rule of thumb the bandwidth of an array with fixed phase shift focusing is the inverse of the time required for the wave to traverse the array. In this case it would imply a bandwidth of the order of 1 Hz, or about 1% of the center frequency. The results of the simulation are shown in Table 2. The overall mean SNR is observed to have fallen by about 3 dB at frequencies 100 \pm 0.5 Hz from what it was at 100 Hz, thus confirming the rule of thumb on bandwidth. The overall mean SNR at 99 Hz has fallen further, the level appearing to be about what one gets when one steers the azimuthal focus away from the source, illuminating the sidelobes.

We point out that the array properties observed here are all normalized to wavelength so that at higher frequencies, with the actual array size reduced but with array size in wavelengths held constant, the bandwidth would remain at about 1%. At 10 kHz we expect a 100 Hz bandwidth, a value adequate for operating a teletype communication link. Furthermore, at this center frequency the array horizontal dimension measured between for points is 15 meters, a dimension one might envision for an array suspended from a surface ship or deployed around a subserged submarine. The array could therefore be useful for underwater data communication. Furthermore, our work was based on focusing by fixed phasing of elements of the array. It is the fixed phasing which limits the array bandwidth. By using controlled time delay networks at each element involved, broader bandwidths are achievable suggesting the possibility of higher speed data communication, or lower center frequency with higher bandwidth.

The multibeam three-dimensional array was compared to four other arrangements as follows:

1. The maximal ratio combiner weighting circuits which multiply each branch output by $V_i^* = A_i e^{-j\psi} i$ where V_i is given by (1) are replaced by constant amplitude phase shifters $e^{-j\psi}i$, thus (5) and (6) become

$$S = \sum_{i=1}^{I} \Lambda_i \tag{8}$$

^{*} The sidelohe properties are discussed further below.

Position	f = 99.0 Hz		f	f = 99.5 Hz		f = 100.5 Hz	
Sample	<snr>_{θφ}</snr>	σ(SNR) _{θφ}	<snr>_⊕ ф</snr>	σ(SNR) _{θφ}	<snr>0¢</snr>	σ(SNR) _{θφ}	
1	156	60	282	112	229	71	
2	147	51	178	45	216	128	
3	167	45	257	42	211	44	
4	236	53	195	72	235	50	
5	206	57	269	117	238	86	
6	244	65	239	189	270	103	
7	195	31	304	104	411	117	
8	172	71	179	25	160	33	
9	229	41	344	57	301	177	
13	138	33	159	42	231	36	
Overall <snr></snr>	189		2	241		250	
σ		64	1	10	1	15	

TABLE 2 SIMULATION OF MRC ARRAY

and

$$N^{2} = \sum_{M=1}^{N} \langle M_{n}^{2} \rangle \sum_{i=1}^{I} \sum_{j=1}^{I} \cos(\phi_{ni} - \phi_{nj} + \psi_{i} - \psi_{j})$$
(9)

Here only phase tracking is needed but the mean SNR will not be as good as for maximal ratio combining. The term "equal gain combining (EGC)" is used in diversity communication for this arrangement.

- 2. The maximal ratio combiner was replaced by a "selection combiner (SC)"; that is, one which simply selects the output with the maximum SNR. This technique is also a standard scheme in communication diversity systems. Whereas the maximal ratio combiner gives an output SNR which is the sum of branch SNRs, this scheme produces only the maximum of the branch SNRs. There is, however, no need for phase tracking, greatly simplifying the processing. It is possible that in applications such as the underwater case the problem is as much variation of vertical arrival angle as it is multipath. In effect then, the array would follow the variation in angle of arrival of the maximum amplitude ray.
- 3. Output was taken from one branch of the multi-beam array, the one which focuses herizontally ($^{\circ}_{si} = 90^{\circ}$). The purpose of this calculation is to see what effect is obtained when the three-dimensional array is operated in its simplest mode.
- 4. The three-dimensional array was reduced to a planar horizontal array. Here we are returning to the original array structure -- the two dimensional array.

Summary results bearing out expectations are shown in Table 3. The maximal ratio combining scheme gives an overall mean SNR of at least 6 dB better than the other arrangements except for EGC case which is a close second. Interestingly, the ratio of c(SNR)/<SNR> is much smaller in the diversity modes than in the two-dimensional case. This too is expected. The diversity modes are partly effective in resolving the multipath and avoiding the non-coherent interference of the multipath components. In the two-dimensional case all rays entering the relatively wide vertical array beanwidth are combined non-coherently. There is, therefore, considerable amplitude variation depending on the relative phases of the accepted rays. In the fixed beam three-dimensional case there is also a high ratio c(SNR)/<SNR> presumably a result of the narrow vertical beamwidth which may or may not see arriving acoustic energy.

An important property of the array will be its response to sources off the azimuth of focus. If we were to imagine swinging the focus away from a source which

	MRC	EGC	sc	90° Sector	2Dimensional
<snr></snr>	480	367	105	32	117
σ(SNR)	182	156	48	30	107

 $\sigma_{\rm u} = 50^{\lambda}$ for all cases

 $h = 25\lambda$ for 3-dimensional arrays

TABLE 3 COMPARISON OF SIMULATION RESULTS

neatly fed M branches independently the power in each branch would on average drop by a factor N, the number of elements. But the final output with MRC or EGC being the coherent sum of the M weighted branches, would only be reduced by a factor $\frac{M}{N}$ from the main beam power. A situation of this sort was simulated to check this surmise. The result obtained using the MRC system was $\langle \text{SNR} \rangle = 182$, the average being over the same set of random variables as before. This figure is somewhat below (M/N) (mainbeam $\langle \text{SNR} \rangle$) but it indicates that these techniques do exact a price in sidelote response. In this calculation there was no signal on the mainbeam.

Further study of sidelobe effects, with and without a main beam signal present and using the different combining schemes, is viewed as a useful next step. In addition, methods based on estimation theoretic principles (e.g., maximum likelihood and maximum entropy estimation) should be considered for application here. These methods inherently maximize on-target signal response relative to off-target signals. Applied to the separate beams as found here, or even to the entire array, superior sidelobe rejection characteristics can be expected.

Fred Haber Paul Yeh

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